Jeremiah Frye and Rob Wojcicki
MTH 222 Lab 01
01/31/02

1) The equation of the curve graphed was $\sigma(t) = (4\cos^3(t), 4\sin^3(t), \cos(at))$. When you scroll through the "a" values of -2 to 2 you see that the curve is not continuous for some values. The graph is continuous whenever the value of "a" is a whole number, such as "a"= 2, 1, 0, -1, -2. An example of "a" equal to 2 is given in figure 1 where you can see that the curve is continuous.

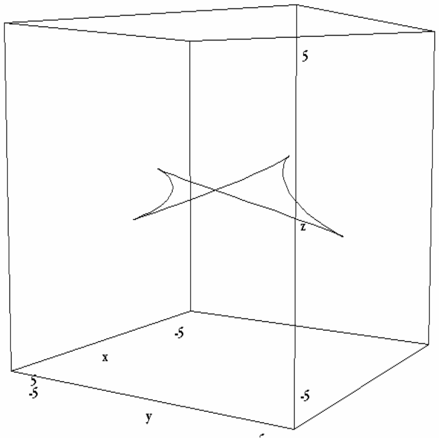

Figure 1: Showing the graph of $\sigma(t) = (4\cos^3(t), 4\sin^3(t), \cos(2t))$, where $0 \le t \le 2\pi$.

When the value of "a" equals these values the corner points of the curve are not smooth. When "a" is equal to 2, 0, or –2 all four corner points are not smooth. When "a" is equal to 1 or -1 two of the corner points are smooth and two are not smooth, an example of this is given in figure 2.

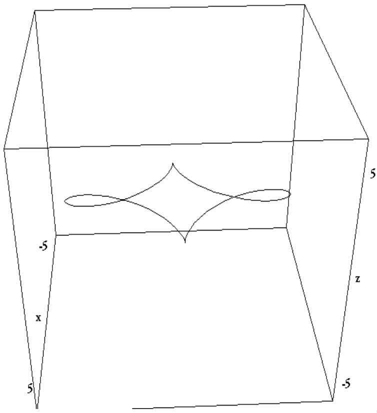

Figure 2: Showing the graph of $\sigma(t) = (4\cos^3(t), 4\sin^3(t), \cos(t))$, where $0 \le t \le 2\pi$.

The graph is only continuous when "a" is equal to a whole number, but this is when the curve has at least two non-smooth corners. This means that all four "corner points" are never smooth at the same time because one of the corners doesn't even exist at certain values of "a". There are four other values of "a" where one corner point is non-smooth and the other two are smooth. These values of "a" are –1.345, 1.345, .65 and -.65. An example of "a" equal to -1.345 is given in figure 3.

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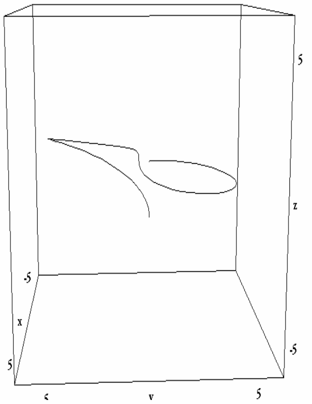

Figure 3: Showing the graph of $\sigma(t) = (4\cos^3(t), 4\sin^3(t), \cos((-1.345)t))$, where $0 \leq t \leq 2\pi$.

All other values of “a” the three corner points are smooth. The following intervals of “a” made the three corner points that did exist smooth: a = (-1.88) - (-1.38), a = (-1.27) - (-1.1), a = (-.87) - (-.78), a = (-.53) - (-.38), a = (.4) - (.52), a = (.74) – (.86), a = (1.1) – (1.27) and a = (1.38) – (1.88). An example showing the curve when all three corner points are smooth is shown in figure 4.

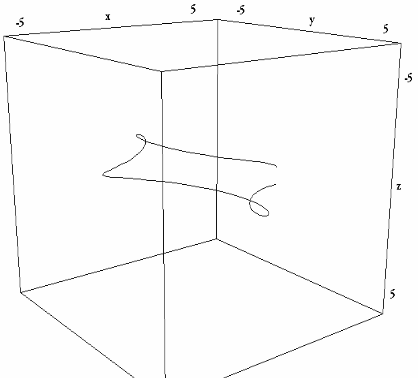

Figure 4: Showing the graph of $\sigma(t) = (4\cos^3(t), 4\sin^3(t), \cos((.796)t))$, where $0 \leq t \leq 2\pi$.

2) The curve obtained from the given equations did not cross itself anywhere, which is shown in figure 5.

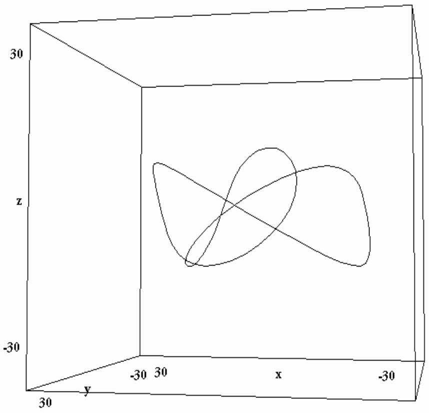

Figure 5: The curve obtained from the equations x(t)= -10cos(t)-2cos(5t)+15sin(2t), y(t)=

$10\sin(t)-2\sin(5t)-15\cos(2t)$, and $z(t)= 10\cos(3t)$, where $0 \le t \le 2\pi$.

At first it looked like the curve might cross itself but when you looked close and moved the curve around you notice that it doesn't cross. To find the points where the curve crossed the 3 standard planes you can graph the individual equations given for x, y, and z on a graphing calculator to determine the values that make the equations equal to zero in the interval 0 to $2\pi$. Once you know the value that makes the one coordinate equal to zero you can plug that value into the other two equations to find the other two coordinates of the point. There were four values of t where the equation of $x(t)$ was equal to zero when t was between 0 and $2\pi$. When $x(t)$ is equal to zero the curve crosses the yz plane. There were also four values of t that made the equation of $y(t)$ equal to zero when t was between the values of 0 and $2\pi$. These points correspond to the curve crossing the xz plane. Last, there were six values of t where the equation of $z(t)$ was equal to zero when t was between 0 and $2\pi$. These points corresponded to the curve crossing the xy plane. The resulting values of t and the coordinates that these values of t gave are summarized in table 1.

Table 1: The different points of intersections of the three standard planes.

| t value | x(t) | y(t) | z(t) |
|---|---|---|---|
| 0.33286 | 0 | -10.52 | 5.415 |
| 1.57 | 0 | 23 | -0.0239 |
| 2.80876 | 0 | -10.52 | -5.4157 |
| 4.7124 | 0 | 7 | 0 |
| 0.6017 | 7.738 | 0 | -2.322 |
| 2.53989 | -7.738 | 0 | 2.322 |
| 4.29688 | 16.864 | 0 | 9.4788 |
| 5.12757 | -16.8566 | 0 | -9.4757 |
| 0.5236 | 6.06 | -3.5 | 0 |
| 1.5708 | 0 | -7 | 0 |
| 2.618 | -6.06 | -3.5 | 0 |
| 3.6652 | 19.9186 | -11.5 | 0 |
| 4.7124 | 0 | 7 | 0 |
| 5.7596 | -19.9185 | -11.5 | 0 |

3) Figure 6 is the graph that was obtained showing the intersection of the two planes.

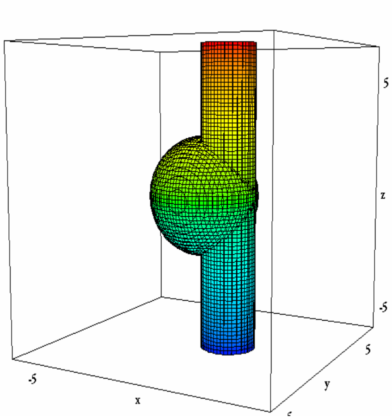

Figure 6: Showing the intersection of the two planes $x^2+y^2+z^2=4$ and $(x-1)^2 +y^2 =1$.

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The parametrization of the curve made by the intersection of these two planes was obtained by doing the following algebraic manipulations on the two original equations of the planes that were given, $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 + z^2 = 4$.

$(x-1)^2 + y^2 = 1$
$x^2 - 2x + 1 + y^2 = 1$
$y^2 = -x^2 + 2x$

We just solved the equation for the cylinder in terms of $y^2$, which can be placed into the equation of the sphere.

$x^2 + y^2 + z^2 = 4$
$x^2 - x^2 + 2x + z^2 = 4$
$2x + z^2 = 4$
$2x = 4 - z^2$
$x = (4 - z^2)/2$

Because we have an equation explicitly in terms of x we can set $z = t$, which makes $x = (4 - t^2)/2$. We can then place these two terms for z and x back into the equation of the sphere to solve for y or we can place the equation for x into the equation of the cylinder and solve for y. You get the same thing either way you do it. The following is what is obtained when the z and x in the equation of the sphere is replaced with t and $(4 - t^2)/2$.

$x^2 + y^2 + z^2 = 4$
$((4 - t^2)/2)^2 + y^2 + t^2 = 4$
$4 - 2t^2 + t^4/4 + y^2 + t^2 = 4$
$y^2 = t^2 - t^4/4$
$y = \pm( t^2 - t^4/4)\hat{}(1/2)$

Notice that the y term is solved in terms of + or -. This means there will be two curves that need to be graphed to get the complete curve of the intersection of the two planes. The graph with the y term negative is shown in figure 7.

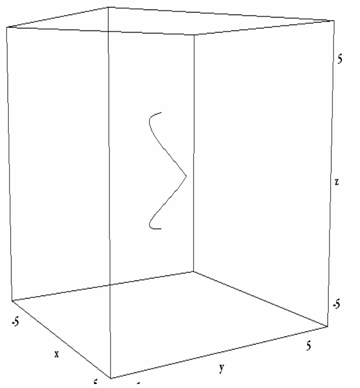

Figure 7: This is the graph of the curve $((4 - t^2)/2, -( t^2 - t^4/4)\hat{}(1/2), t)$.

The graph of the two curves graphed at the same time is shown in figure 8.

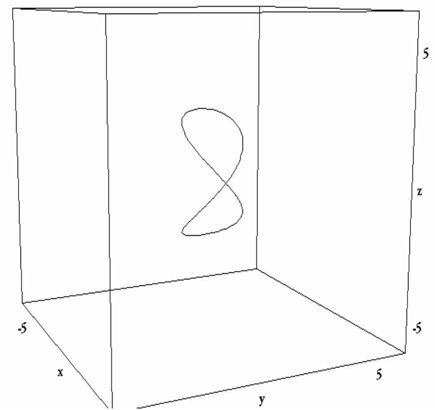

Figure 8: This is the graph of the curves $((4-t^2)/2, +(t^2-t^4/4)^\wedge(1/2), t)$ and $((4-t^2)/2, -(t^2-t^4/4)^\wedge(1/2), t)$.

4) We graphed the function y=2e^-(x/2)sin(ax) and set “a” equal to 1, 2, and 3. The graphs that were obtained are shown in figures 9, 10, and 11, respectively.

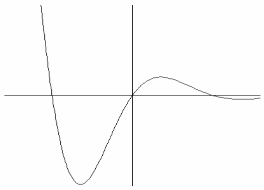

Figure 9: Showing the graph of the function y=2e^-(x/2)sin(x).

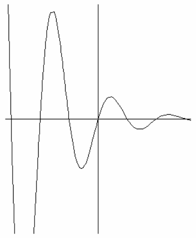

Figure 10: Showing the graph of the function y=2e^-(x/2)sin(2x).

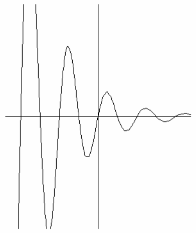

Figure 11: Showing the graph of the function y=2e^-(x/2)sin(3x).

The curve that we would approach if we allowed the value of “a” to be as large as possible would be $y = 0$. The reason this is so is because as “a” gets bigger and bigger the wavelength decreases, but at the same time the amplitude of the waves are getting smaller and smaller. As the value of “a” gets larger the amplitudes get closer to $y = 0$. The graph of “a” = 39.92 is shown in figure 12.

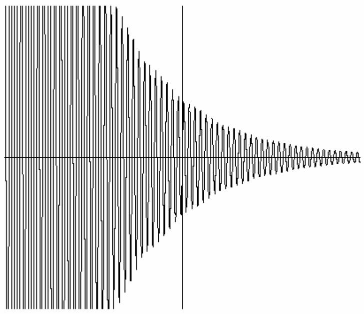

Figure 12: Showing the graph of the function y=2e^-(x/2)sin(39.92x).

If we allowed the value of “a” to go to zero the equation will go to zero because the sin of zero is zero. So the equation of the line that the function would approach as the value of “a” goes to zero will be $y = 0$ also. The graph of a = .04 is shown in figure 13.

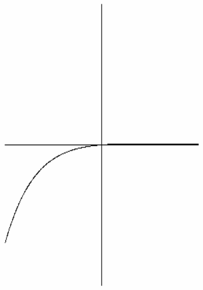

Figure 13: Showing the graph of the function y=2e^-(x/2)sin(.04x).

The graph of a = .02 is shown in figure 14.

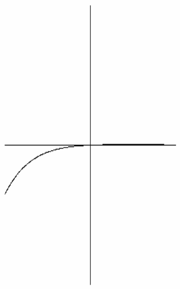

Figure 14: Showing the graph of the function y=2e^-(x/2)sin(.02x).

You can see the line approaches $y = 0$ as the value of “a” gets smaller and smaller.

5) From the graph of the equation that is given you can see that the bounded piece of the curve is a closed circle. However it is not a perfectly round circle. The graph of this function is given in figure 15.

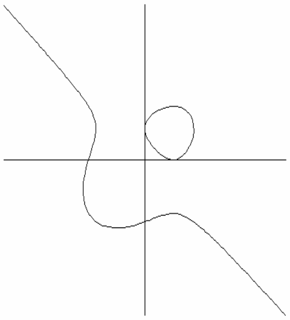

Figure 15: The graph obtained from the function $3(x+y)-x^3-y^3=2$.

To find the coordinates of these points you can set one of the variables equal to zero in the equation and then solve for the other one. By doing this you find the values of the intersections. The following is how to solve for the intersection points.

$3(x+y)-x^3-y^3 = 2$
$3x+3y-x^3-y^3 = 2$
$3x - x^3-2 = y^3-3y$ set x equal to zero and you get
$-2+3y = y^3$

The only values that satisfy this equation is when $y = 1$ or $–2$. So the points that you get for the intersection of the y-axis is (0, 1) and (0, -2). For the x-axis the process is the same only you set y equal to zero instead of x. By doing this you get $–2 = x^3-3x$. The only values that satisfy this equation are 1 and –2. The points of intersection of the x-axis from these values gives (1, 0) and (-2, 0).